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SE-6121

B. E. II (Sem. III) (EC/EL/CHEM) Examination

April / May – 2011

Engg. Maths - III

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. E. 2 (Sem. 3) (EC/EL/CHEM)**

Name of the Subject :
☛ **Engg. Maths - III**

☛ Subject Code No. : **6 1 2 1** ☛ Section No. (1, 2,.....) : **1&2**

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Figures to the right indicate marks.

SECTION - I

1 (a) Do as directed : **10**

(i) Express the integral

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$$

by changing the order of integration.

(ii) Express the integral

$$\int_0^\infty \int_x^\infty e^{-\left(x^2+y^2\right)}$$ dxdy in the polar form.

(iii) Define divergence of a vector. Find div $\bar{r}$ where $\bar{r}$ is the position vector of p(x,y,z)

(iv) Find unit normal vector to the plane x+2y+3z=4.

(v) f(x), in interval $[-\pi, \pi]$ is define as

$$f(x)=\begin{cases} 1+\dfrac{2x}{\pi}, & if -\pi \le x \le 0 \\ 1-\dfrac{2x}{\pi}, & if\ 0 \le x \le \pi \end{cases}$$

Is f(x) an even function, or an odd function or none ?

(b) Attempt any three : **12**

(1) Evaluate

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$$

(2) Calculate $\iint r^3 dr\, d\theta$ over the area included between the circles $r=2\sin\theta$ and $r=4\sin\theta$

(3) Find the volume common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$.

(4) Find the mass of the tetrahedron bounded by the co-ordinate planes and the plane x+y+z=1, if the variable density $\rho=\mu xyz$, where μ is a constant.

2 (a) Attempt any two : **06**

(i) Find the directional derivative of $f(x,y,z)=xy^2+yz^3$ at the point (2, –1, 1) in the direction of vector $\hat{i}+2\hat{j}+2\hat{k}$

(ii) Show that

$$\nabla^2(r^n)=n(n+1)r^{n-2}$$

(iii) Show that

$$\bar{F}=2xyz\hat{i}+(x^2z+2y)\hat{j}+x^2y\hat{k}$$

is irrotational. Find its scalar potential function also.

(b) Attempt any two : **08**

(i) Applying Green's theorem, evaluate

$\int_C [(y-\sin x)\,dx+\cos x\,dy]$ where C is the plane triangle enclosed by the lines y = 0, $x=\pi/2$ and $y=2x/\pi$

(ii) Evaluate by Stoke's theorem

$$\oint_C (yzdx+zxdy+xydz)$$

where C is the curve $x^2+y^2=1, z=y^2$

(iii) Using divergence theorem,

prove that $\int_s \nabla r^2 . d\bar{s}=6v$

where s is any closed surface enclosing a volume V and $r^2=x^2+y^2+z^2$

3 (a) Find the fourier sine series for **4**

$f(x) = \pi x - x^2 \ in \ (0, \pi)$

(b) Attempt any two : **10**

(i) Obtain a Fourier series to represent e^{-ax} from $x = -\pi \ to \ x = \pi$

(ii) Find the Fourier series expension for f(x), if

$f(x) = -\pi, \ -\pi < x < 0$

$= x, \ 0 < x < \pi$

(iii) Find a fourier series for

$f(x) = \pi x \ \ in \ 0 \le x \le 2$

SECTION - II

4 (a) Do as directed : **10**

(i) Define Gamma function.

Derive $\Gamma(n+1) = n\Gamma(n), \ n > 0$

(ii) Define error function.

Show that $erf(-x) = -erf(x)$

(iii) Write one-dimensional heat equation. Also write its physically acceptable solution.

(iv) What can you say about analyticity of function $w = \log z$.

(v) State Cauchy integral theorem.

(b) Attempt any two : **06**

(i) Show that

$$\int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx = \Gamma(n),\ n > 0$$

(ii) Show that

$$\int_0^1 \frac{dx}{\sqrt{1-x^{1/4}}} = \frac{128}{35}$$

(iii) Show that

$$\int_0^\infty \frac{x^{3/2}}{(1+x)^5}\, dx = \frac{3\pi}{128}$$

(c) Solve any two : **06**

(i) $\frac{y^2 z}{x} p + xzq = y^2$

(ii) $pz - qz = z^2 + (x+y)^2$

(iii) $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

5 Attempt any two : **12**

(i) An insulated rod of length ℓ has its ends A and B maintained at 0^0C. and 100^0 C respectively, until steady state conditions prevail. If B is suddenly reduced to 0^0C and maintained at 0^0C, find the temperature at a distance x from A at time t.

(ii) A tightly stretched string with fixed end points x=0 and x = ℓ, is initially at rest in its equilibrium position. If it is set vibrating bygiving to each of its points a velocity $\lambda x(\ell - x)$, find the displacement of the string at any distance x from one end at any time t.

(iii) Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ for $0<x<\pi,\ 0<y<\pi$, with conditions given :

$u(0,y)=u(\pi,y)=u(x,\pi)=0$

and $u(x,0)=\sin^2 x$

6 (a) Obtain Cauchy-Riemann equation in polar form. **4**

Also show that, if $w=u(r,\theta)+i\,v(r,\theta)$ is an analytic function then $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$

(b) Attempt any two : **06**

(i) Show that image of the hyperbola $x^2-y^2=1$ is the lemniscate $\rho^2=\cos 2\phi$ under the transformation $w=\frac{1}{z}$

(ii) Determine the analytic function whose real part is $\frac{1}{2}\log\left(x^2+y^2\right)$

(iii) Find the bilinear transformation that maps the points $z=-1,0,1$ onto $w=-1,-i,1$ respectively.

(c) Solve any two : **06**

(i) Evaluate $\oint_c \frac{e^z}{(z+1)^2} dz$ where C is the circle $|z|=3/2$

(ii) Evaluate

$$\oint_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz, \text{ where}$$

C is the circle $|z|=3$

(iii) Evaluate

$$\oint_c \frac{z^2+1}{z^2-1} dz, \text{ where}$$

C is the circle $|z|=1/2$.